# Optimal Design of Reinforced Concrete Box Culvert by Using Genetic Algorithms Method 

Dr. Abdul-Hassan K. Al. Shukur1, Dr. Mohammed Abbas Al. Jumaili2, Hawraa Ali Hussein3


#### Abstract

This paper shows the optimal design of reinforced concrete box culvert based on minimum cost (economical design). The Genetic Algorithms (GAs) is the proposed method to optimize the structure which used as a tools box in MATLAB software version 2011, and the results of these method were verified by using Sequential Quadratic Programming (SQP) method which also used as tool box in MATLAB software version 2011. The formulation of the problem includes 11 design variables: two geometrical, and 9 structural variables for the reinforcement set-up and the thickness of top, bottom, and the thickness of side walls. A parametric study was conducted to specify initial population and population size and concluded that the optimum results were obtain for initial population $=\left[\begin{array}{ll}1.8 & 1.8 \\ 0.2 & 0.0001 \\ 0.0001\end{array}\right.$ $0.00010 .00010 .00010 .00010 .0002327737280 .000232773728]$, and population size=100.


Index Terms - Box Culvert, Genetic Algorithms MATLAB, Optimal Design, Population.

## 1 Introduction

The present design of economical concrete structures mainly follows rules based on the experience of structural engineers. Most procedures adopt the cross-section dimensions and material grades sanctioned by common practice. Once the structure is defined according to experience, it is necessary to analyze the stress resultants and compute the positive and negative reinforcement so as to satisfy the limit states prescribed by concrete codes. If the initial design dimensions or material grades be insufficient or excessive, the structure is redefined on a trial-and-error basis. Such a process leads to safe designs, but the cost of the concrete structures is highly dependent on the experience of the structural engineer. Moreover, these designs lack objectivity in the sense that different designers obtain different results despite adhering to the same concrete codes. Structural optimization methods are clear alternatives to designs based on experience [1].

## 2. OPTIMIZATION PROBLEM DEFINITION

### 2.1 DESIGN VARIABLES

The design variables are the width of the reinforced concrete box culvert ${ }_{1}$, rise of the reinforced concrete box culvert $X_{2}$, thickness of top and bottom slabs and the thickness of the side walls ${ }_{3}$, area steel required to resist negative moment in top slab (upper layer) ${ }_{4}$, area

[^0]steel required to resist positive moment in top slab (lower layer) ${ }_{5}$, area steel required to resist negative moment in side wall (outer layer) ${ }^{X_{6}}$, area steel required to resist positive moment in side wall (inside layer) $X_{7}$, area steel required to resist negative moment in bottom slab (outside layer) ${ }^{X_{8}}$, area steel required to resist positive moment in bottom slab (inside layer) ${ }^{X_{9}}$,shrinkage and temperature area steel in top and bottom slabs $X_{10}$, and shrinkage and temperature area steel in side walls $X_{11}$, all these variables are shown in Fig. 1 below.


Fig. 1. Typical section of reinforced concrete box culvert

### 2.2 OBJECTIVE FUNCTION

The problem of structural concrete optimization proposed in this study consists of an economic optimization. It deals with the minimization of the objective function $f(X)$ of expression below, satisfying as well the constraints of

$$
g_{2}=\left(1+K_{E}+\frac{2 g n^{2} L}{\left(\frac{x_{1} x_{2}}{2\left(x_{1}+x_{2}\right)}\right)^{4 / 3}}\right) \frac{Q^{2}}{2 g x_{1}^{2} x_{2}^{2}}-H \leq 0
$$ section 2.3.

$f(X)=\left(x_{1}+2 x_{3}\right)\left(x_{2}+2 x_{3}+D_{f}\right) U_{e x}+\left(\left(x_{1}+2 x_{3}\right)\left(x_{2}+2 x_{3}\right) 2.3 x_{13} x_{2}\right)$ Besign velocity at the peak design discharge $+\left(x_{1}+2 x_{3}\right) D_{f} \mathrm{x} U_{f}+\left(x_{1}+2 x_{3}\right) x_{4} \rho_{s} U_{s}+\left(x_{1}+2 x_{3}\right) x_{5} \rho_{s} U_{\text {rate determined from the hydrological analysis in }}$ $+2\left\{\left(x_{2}+2 x_{3}\right) x_{6}+\left(x_{2}+2 x_{3}\right) x_{7}\right\} \rho_{s} U_{s}+\left(x_{1}+2 x_{3}\right) x_{8} \rho_{s} U_{s}$ the culvert shall be greater than 1 meter/second $\left.2 x_{3}\right) x_{9} \rho_{s} U_{s}+4 x_{10} \rho_{s} \times U_{s}+4 x_{11} \rho_{s} \times U_{s}$
$U_{e x}=$ unit cost of excavation, (unit price $/ m^{3}$ )
$g_{3}=1-\frac{Q}{x_{1} x_{2}} \leq 0$
$U_{c}=$ unit cost of concrete (labor and material), ( unit price $/ \mathrm{m}^{3}$ )
$U_{f}=$ unit cost of earth fill, (unit price / $m^{3}$ )
2.3.1.4 When velocities exceed about $3(\mathrm{~m} / \mathrm{s})$, abrasion due to bed movement through the culvert and erosion downstream of outlet can increase significantly [5].
$U_{s}=$ unit cost of steel reinforcement (labor and material), ( unit price / ton )
$\rho=$ density of steel reinforcement, (Ton $/ \mathrm{m}^{3}$ )

The cost function is the value of materials and all the entries required to evaluate the entire cost of the reinforced concrete box culvert per linear meter (formwork, excavation, fill, etc). The prices considered were provided by local contractors of road construction in October 2012.

### 2.3 CONSTRAINTS

The objective function constraints used in this study is applied to ensure that the reinforced concrete box culvert constructed with minimum cost will accomplished all the necessary requirements required to performed its function perfectly thus these constraints are:

### 2.3.1 HYDRAULIC CONSTRAINTS

2.3.1.1 Where inlet control exits, the head required at culvert inlet is computed from the orifice equation [2].

$$
g_{1}=\frac{Q^{2}}{2 g C^{2}\left(x_{1} x_{2}\right)^{2}}-1.2 x_{2} \leq 0
$$

2.3.1.2 To evaluate the outlet control hydraulics the condition of full flow in the culvert barrel will be used. The energy equation must incorporate the losses due to entrance ( ${ }^{h_{o}}$ ), friction ( ${ }^{h_{f}}$ ), exit ( $h_{e x}$ ), and can be written as [3]:

$$
g_{4}=\frac{Q}{x_{1} x_{2}}-3 \leq 0
$$

### 2.3.2 STRUCTURAL CONSTRAINT

2.3.1.5 Box culverts and frames with clear span to rise ratios that exceed 4 are not recommended [6].

$$
g_{5}=\frac{x_{1}}{x_{2}}-4 \leq 0
$$

2.3.1.6 Slab thickness shall be equal to the following equation for crack control criteria [7].

$$
g_{6}=0.0102\left(3.3 x_{1}+10\right)-x_{3} \leq 0
$$

2.3.1.7 The primary reinforcing steel required to resist negative moment in top slab (upper layer) can be calculated as:

$$
g_{7}=\frac{0.85 f_{c}^{\prime} b d}{f_{y}}\left(1-\sqrt{1-\frac{4 M_{r_{1}}}{1.53 f^{\prime} c b d^{2}}}\right)-x_{4} \leq 0
$$

$M_{r_{1}}=$ negative moment in top slab of reinforced concrete box culvert.
2.3.1.8 The primary reinforcing steel shall be adequate to develop a factored flexural resistance, Mr , at least equal to the lesser of 1.2 times the cracking moment, Mcr and 1.33 times the factored moment required by the applicable strength load combinations.

$$
M_{r}=\min \left(1.2 M_{c r}, 1.33 M_{u}\right)
$$

$$
\begin{aligned}
& \text { if } M_{r}=1.2 M_{c r} \\
& g_{8}=\frac{0.85 f_{c}^{\prime} b d}{f_{y}}\left(1-\sqrt{1-\frac{148 x_{3}^{2}}{765 \sqrt{f_{c}^{\prime} d^{2}}}}\right)-x_{4} \leq 0
\end{aligned}
$$

Else if $M_{r}=1.33 M_{u}$

$$
g_{8}=\frac{0.85 f_{c}^{\prime} b d}{f_{y}}\left(1-\sqrt{1-\frac{5.32 M_{r_{1}}}{1.53 f_{c}^{\prime} b d^{2}}}\right)-x_{4} \leq 0
$$

2.3.1.9 The provision for maximum reinforcement according to the LRFD deign method for reinforced concrete box culvert is deleted since 2005 [7], but for optimization technique the maximum value of steel reinforcement must be specified, therefore the maximum steel reinforcement will be specified depending on the provision of [8].

$$
\begin{aligned}
g_{9} & =x_{4}-\left(0.428 * 0.85 \beta_{1} \frac{f^{\prime} c d_{t}}{f y d}\right) \leq 0 \\
g_{10} & =\frac{0.85 f_{c}^{\prime} b d}{f_{y}}\left(1-\sqrt{1-\frac{4 M_{r_{2}}}{1.53 f^{\prime} b d^{2}}}\right)-x_{5} \leq 0
\end{aligned}
$$

2.3.2.1 The primary reinforcing steel required to resist positive moment in top slab (lower layer) shall satisfy the following constraints:
$M_{r_{2}}=$ positive moment in top slab of reinforced concrete box culvert.

$$
\begin{aligned}
& \text { for } M_{r}=1.2 M_{c r} \\
& g_{11}=\frac{0.85 f_{c}^{\prime} b d}{f_{y}}\left(1-\sqrt{1-\frac{148 \mathrm{t}^{2}}{765 \sqrt{f_{c}^{\prime}} d^{2}}}\right)-x_{5} \leq 0 \\
& \text { for } M_{r}=1.33 M_{u} \\
& g_{11}=\frac{0.85 f_{c}^{\prime} b d}{f_{y}}\left(1-\sqrt{1-\frac{5.32 M_{r_{2}}}{1.53 f_{c}^{\prime} b d^{2}}}\right)-x_{5} \leq 0 \\
& g_{12}=x_{5}-\left(0.428 * 0.85 \beta_{1} \frac{f^{\prime} d_{t}}{f y d}\right) \leq 0
\end{aligned}
$$

2.3.2.2 The primary reinforcing steel required to resist negative moment in side wall (outer layer) shall satisfy the following constraints:
$g_{13}=\frac{0.85 f_{c}^{\prime} b d}{f_{y}}\left(1-\sqrt{1-\frac{4 M_{r_{3}}}{1.53 f^{\prime}{ }^{\prime} b d^{2}}}\right)-x_{6}$
$M_{r_{3}}=$ negative moment in exterior side wall of reinforced concrete box culvert.

$$
\begin{aligned}
& \text { for } M_{r}=1.2 M_{c r} \\
& g_{14}=\frac{0.85 f_{c}^{\prime} b d}{f_{y}}\left(1-\sqrt{1-\frac{148{x_{3}}^{2}}{765 \sqrt{f_{c}^{\prime} d^{2}}}}\right)-x_{6} \leq 0 \\
& \text { for } M_{r}=1.33 M_{u} \\
& g_{14}=\frac{0.85 f_{c}^{\prime} b d}{f_{y}}\left(1-\sqrt{1-\frac{5.32 M_{r_{3}}}{1.53 f_{c}^{\prime} b d^{2}}}\right)-x_{6} \leq 0 \\
& g_{15}=x_{6}-\left(0.428 * 0.85 \beta_{1} \frac{f_{c}^{\prime} d_{t}}{f y d}\right) \leq 0
\end{aligned}
$$

2.3.2.3The primary reinforcing steel required to resist positive moment in side wall (inner layer) shall satisfy the following constraints:
$g_{16}=\frac{0.85 f_{c}^{\prime} b d}{f_{y}}\left(1-\sqrt{1-\frac{4 M_{r_{4}}}{1.53 f^{\prime} c b d^{2}}}\right)-x_{7} \leq 0$
$M_{r_{4}}=$ positive moment in exterior side wall of reinforced concrete box culvert
for $M_{r}=1.2 M_{c r}$
$g_{17}=\frac{0.85 f_{c}^{\prime} b d}{f_{y}}\left(1-\sqrt{1-\frac{148 \mathrm{t}^{2}}{765 \sqrt{f_{c}^{\prime}} d^{2}}}\right)-x_{7} \leq 0$
for $M_{r}=1.33 M_{u}$
$g_{17}=\frac{0.85 f_{c}^{\prime} b d}{f_{y}}\left(1-\sqrt{1-\frac{5.32 M_{r_{4}}}{1.53 f_{c}^{\prime} b d^{2}}}\right)-x_{7} \leq 0$

$$
g_{18}=x_{7}-\left(0.428 * 0.85 \beta_{1} \frac{f^{\prime} c d_{t}}{f y d}\right) \leq 0
$$

2.3.2.4The primary reinforcing steel required to resist negative moment in bottom slab (lower layer) shall satisfy the following constraints:

$$
\begin{aligned}
& g_{19}=\frac{0.85 f_{c}^{\prime} b d}{f_{y}}\left(1-\sqrt{1-\frac{4 M_{r_{5}}}{1.53 f^{\prime} c d^{2}}}\right)-x_{8} \\
& M_{r_{5}}=\text { negative moment in bottom slab of }
\end{aligned}
$$ reinforced concrete box culvert.

$$
\begin{aligned}
& \text { for } M_{r}=1.2 M_{c r} \\
& g_{20}=\frac{0.85 f_{c}^{\prime} b d}{f_{y}}\left(1-\sqrt{1-\frac{148 \mathrm{t}^{2}}{765 \sqrt{f_{c}^{\prime}} d^{2}}}\right)-x_{8} \leq 0
\end{aligned}
$$

for $M_{r}=1.33 M_{u}$
$g_{20}=\frac{0.85 f_{c}^{\prime} b d}{f_{y}}\left(1-\sqrt{1-\frac{5.32 M_{r_{2}}}{1.53 f_{c}^{\prime} b d^{2}}}\right)-x_{8} \leq 0$
$g_{21}=x_{8}-\left(0.428 * 0.85 \beta_{1} \frac{f^{\prime}{ }_{c} d_{t}}{f y d}\right) \leq 0$
2.3.2.5primary reinforcing steel required to resist positive moment in bottom slab (lower layer) can

$$
g_{22}=\frac{0.85 f_{c}^{\prime} b d}{f_{y}}\left(1-\sqrt{1-\frac{4 M_{r_{6}}}{1.53 f^{\prime} c b d^{2}}}\right)-x_{9} \leq 0
$$

be calculated as:

$$
M_{r_{6}=\text { negative }} \text { moment in bottom lab of }
$$ reinforced concrete box culvert

$$
\begin{aligned}
& \text { for } M_{r}=1.2 M_{c r} \\
& g_{23}=\frac{0.85 f_{c}^{\prime} b d}{f_{y}}\left(1-\sqrt{1-\frac{148 \mathrm{t}^{2}}{765 \sqrt{f_{c}^{\prime}} d^{2}}}\right)-x_{9} \leq 0 \\
& \text { for } M_{r}=1.33 M_{u}
\end{aligned}
$$

$$
\begin{aligned}
& g_{23}=\frac{0.85 f_{c}^{\prime} b d}{f_{y}}\left(1-\sqrt{1-\frac{5.32 M_{r_{6}}}{1.53 f_{c}^{\prime} b d^{2}}}\right)-x_{9} \leq 0 \\
& g_{24}=x_{9}-\left(0.428 * 0.85 \beta_{1} \frac{f^{\prime}{ }_{c} d_{t}}{f y d}\right) \leq 0
\end{aligned}
$$

2.3.2.6 For shrinkage and temperature reinforcement in top and bottom slabs the following constraints must be satisfied:
$g_{25}=\frac{1.30 b x_{3}}{2\left(x_{1}+3 x_{3}\right) f_{y}}-x_{10} \leq 0$
$g_{26}=0.11-x_{10} \leq 0$
$g_{27}=x_{10}-0.6$
2.3.2.7 The constraints for shrinkage and temperature reinforcement in side walls can be stated by using the following equations:
$g_{28}=\frac{1.30 b x_{3}}{2\left(x_{2}+3 x_{3}\right) f_{y}}-x_{11} \leq 0$
$g_{29}=0.11-x_{11} \leq 0$
$g_{30}=x_{11}-0.6$
2.3.2.8The shear resistance in top and bottom slabs must satisfy the following constraints for 2.0 ft or more of fill:
$g_{28}=\operatorname{Vud}_{(\text {top slab })}-\phi\left(0.0676 \sqrt{f_{c}{ }^{\prime}} b d_{e}\right) \leq 0$
$g_{29}=\operatorname{Vud}_{(\text {bottom slab })}-\phi\left(0.0676 \sqrt{f_{c}{ }^{\prime}} b d_{e}\right) \leq 0$
2.3.2.9 The shear resistance of concrete in side walls shall satisfy the following constraint for 2.0 ft or more of fill:
$g_{30}=\operatorname{Vud}_{(\text {side wall) }}-0.0316 \beta \sqrt{f_{c}{ }^{\prime}} b_{v} d_{v} \leq 0$
2.3.2.10 For box culverts with less than 2.0 feet of fill. The shear resistance of the concrete, Vc, for slabs and walls of box culverts shall satisfy the following constraints:
$g_{28}=V u d_{(\text {top slab })}-0.0316 \beta \sqrt{f_{c}{ }^{\prime}} b_{v} d_{v} \leq 0$

$$
\begin{aligned}
& g_{29}=\operatorname{Vud}_{(\text {bottom slab ) }}-0.0316 \beta \sqrt{f_{c}^{\prime}} b_{v} d_{v} \leq 0 \\
& g_{30}=\operatorname{Vud}_{(\text {side wall) })}-0.0316 \beta \sqrt{f_{c}^{\prime}} b_{v} d_{v} \leq 0
\end{aligned}
$$

### 2.3.3 GEOTECHNICAL CONSTRAINTS

2.3.3.1 The gross allowable load-bearing capacity of shallow foundations requires the application of a factor of safety (FS) to the gross ultimate bearing capacity [9] or
$q_{\text {all }}=\frac{q_{u}}{F s}$
$q_{u} \geq q_{\text {all }} F s$
$q_{\text {all }}=\gamma_{f} D_{f}+\gamma_{c}\left(\left(x_{1}+2 x_{3}\right)\left(x_{2}+2 x_{3}\right)-x_{1} x_{2}\right) /\left(x_{1}+2 x_{3}\right)+\gamma_{w} x_{2}$

$$
q_{u}=c^{\prime} N_{c} F_{c s} F_{c d}+q N_{q} F_{q s} F_{q d}+\frac{1}{2} \gamma_{s} B N_{\gamma} F_{\gamma s} F_{\gamma d}
$$

2.3.3.5 the minimum thickness of reinforced concrete box culvert should be more than $8^{\prime \prime}(0.2 \mathrm{~m})$
$g_{36}=0.2-x_{3} \leq 0$
2.3.3.6 the following side constraints are specified for better performance of the optimization technique
$g_{37}=x_{2}-4 \leq 0$
$g_{38}=x_{2}-x_{1} \leq 0$
$g_{39}=x_{3}-0.6 \leq 0$

## 3. GENETIC ALGORITHMS PROCESS [13]

An algorithm is a series of steps for solving a problem. A GA is a problem solving method that uses genetics as its model of problem solving. It's a search technique to find approximate solutions to optimization and search problems. The basic steps of GAs process are [14]:

Step(1): Creation Initial Population: Genetic
 $\gamma_{f}\left(x_{2}+2 x_{3}+D_{f}\right) N_{q} F_{q s} F_{q d}+\frac{1}{2} \gamma_{s}\left(x_{1}+2 x_{3}\right) N_{\gamma} F_{\gamma s} F_{\gamma d} \leq 0$
2.3.3.2 More than $70 \%$ of the soil type in Al -Najaf city is sand soil in both the old and the new extension of Al-Najaf city with angle of internal friction ranging from $(30-35)$ in old city and from (35-40) in the new extension of the city [10]. Therefore the total settlement will be equal to the elastic settlement only when the reinforced concrete box culvert is proposed to be constructed in Al- Najaf city.

Step(2): Fitness Evaluation: Evaluate the fitness $f(x)$ of each chromosome $x$ in the population.

Step(3): (Breeding): Create a new population by repeating following steps until the new population is complete.

Selection: Select two parent chromosomes from a population according to their fitness (the better fitness, the bigger chance to get selected).

Crossover : With a crossover probability, cross over the parents to form new offspring (children). If no crofsover ${ }_{\mu_{s}}^{2}$ has performed, offspring is the exact copy of


### 2.3.4 SIDE CONSTRAINTS

2.3.3.3 Minimum box dimension shall be 0.9 by 0.9 m [11].
$g_{33}=0.9-x_{1} \leq 0$
$g_{34}=0.9-x_{2} \leq 0$
2.3.3.4 Four sided boxes can typically be used for spans up to $12 \mathrm{ft}(3.5 \mathrm{~m})$ [12].
$g_{35}=x_{1}-3.5 \leq 0$

Mutation: With a mutation probability, mutate new offspring at each locus (position in chromosome)

Replacement : Use new generated population for a further sum of the algorithm.

Step(4): Condition Criteria: If the end condition is satisfied, stop, and return the best solution in current population.

Step(5): Cycling Operation (Loop) Go to step (2) for fitness evaluation.

## 4. APPLICATION AND VERIFICATION

In order to show the application of optimization solution using the objective function, constraints, and input data of material properties using genetic algorithm solver with MATLAB software the following example will be used as: Design the reinforced concrete box culvert to pass the design discharge of value equal to $\left(12 \mathrm{~m}^{3} / \mathrm{sec}\right)$ with Headwall parallel to embankment (no wingwalls) (Squareedged on 3 edges) with coefficient of entrance loss coefficient equal to ( $K_{E}=0.5$ ) and total losses in culvert barrel equal to ( $\mathrm{H}=0.15$ ), The length of box culvert equals to ( $\mathrm{L}=26.75 \mathrm{~m}$ ).The depth of fill above the top slab of culvert equals to ( $D_{f}=1 m$ ), the properties of materials (concrete, steel, foundation soil, and fill soil) and the price of material can be shown in Table 1.

Table 1 the data of design example

| Material | Symbol | Value | Unite |
| :--- | :--- | :--- | :--- |
| Concrete | $\gamma_{c}$ | 24 | $\mathrm{kN} / \mathrm{m}^{3}$ |
|  | $f_{c}^{\prime}$ | 30 | MPa |
|  | n | 0.012 | Dimension less |
| Steel | $\rho_{s}$ | 7.85 | Ton $/ \mathrm{m}^{3}$ |
|  | $f_{y}$ | 420 | MPa |
| Foundation <br> soil | $\gamma_{s}$ | 20 | $\mathrm{kN} / \mathrm{m}^{3}$ |
|  | $\phi_{s}$ | 35 | Degree |
| Fill soil | $\gamma_{f}$ | 18.5 | $\mathrm{kN} / \mathrm{m}^{3}$ |
|  | $\phi_{f}$ | 30 | Degree |
| Price of <br> concrete | $P_{c}$ | 145 | U.P. |
| Price of <br> steel | $P_{s}$ | 1100 | U.P. |
| Price of fill <br> soil | $P_{f}$ | 9.5 | U.P. |
| Price of <br> excavation | $P_{e x}$ | 4 | U.P. |

### 4.1 RESULT OF APPLICATION OF OPTIMAZATION PROCESS

The solution of the example stated above (in Application and verification) was found as shown in Table 2:

Table 2 Results of Design Example

| Deign variable | Symbol | Variable value |
| :--- | :--- | :--- |
| Culvert clear width | $x_{1}$ | 2.98 m |
| Culvert clear height | $x_{2}$ | 2.98 m |
| Thickness of top. bottom <br> slab. and side walls | $x_{3}$ | 0.20 m |
| Steel reinforcement in top <br> slab(outer layer) | $x_{4}$ | 1796.10 mm |
| Steel reinforcement in top <br> slab(inner layer) | $x_{5}$ | 690.50 m |
| Steel reinforcement in side <br> wall (outer layer) | $x_{6}$ | 1796.10 mm |
| Steel reinforcement in side <br> wall (inner layer) | $x_{7}$ | 1165.33 mm |
| Steel reinforcement in <br> bottom slab (outer layer) | $x_{8}$ | 953.22 mm |
| Steel reinforcement in <br> bottom slab (inner layer) | $x_{9}$ | 338.88 mm |
| Shrinkage and temperature <br> reinforcement in top and <br> bottom slab | $x_{10}$ | 232.77 mm |
| Shrinkage and temperature <br> reinforcement in side walls | $x_{11}$ | 232.77 mm |
| Total price | fval | $764.57 \mathrm{U.P}$. |

In this work, a detailed study of optimum design of reinforced concrete box culvert was carried out without initial population. Ten randomly runs using MATLAB software for each population size was utilized to study the effects of begin with randomly initial population and different population size. Results of these ten runs for each population size are showed in Table 3. Comparing these results with optimum design solution as shown in Table 2 indicates that the number of correct results is only one out of ten runs for each population size. The number of correct results is the counter of runs in which the correct value of objective function (fval=764.57) is obtained. Also as the population size is increased there is no noticeable improvement in the number of correct results and the elapse time of each run increases especially for population size more than 100. The comparison of optimum values for constant population size (i.e. Pop. size $=100$ ) can be shown in Fig. 2. It appears that the value of objective function of the first generation for each run is different due to randomly initial population. Therefore, it can be concluded that the case of GAs optimization without initial population is unsuitable to the problem under
consideration and it seems necessary to specify the initial population（i．e．initial point）．
TABLE 3：COMPARISON OF RESULTS FOR VARIOUS POPULATION SIZE WITHOUT INITIAL POPULATION
＊Best Results
Comparing these results with optimum design

| $\begin{aligned} & \text { N } \\ & \text { W } \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | Objective function（Total Price）U．P． |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | N | $$ |  |  |  |  | 为 |  | $\begin{aligned} & \text { o } \\ & 0 \\ & 0 \\ & \vdots \end{aligned}$ |
| 은 | $\begin{aligned} & \hat{\alpha} \\ & \dot{\theta} \end{aligned}$ | $\begin{aligned} & \text { กొ } \\ & \stackrel{1}{\Omega} \end{aligned}$ | $\begin{aligned} & \text { N} \\ & \text { O. } \\ & \text { in } \end{aligned}$ | $\begin{aligned} & 0 \\ & \text { i } \\ & \mathbb{Z} \end{aligned}$ | $\begin{aligned} & 10 \\ & \stackrel{10}{3} \\ & \stackrel{\rightharpoonup}{\sigma} \end{aligned}$ | $\begin{aligned} & \text { İ } \\ & \underset{\sim}{\infty} \\ & \text { Nָ } \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \text { © } \\ & \stackrel{\rightharpoonup}{\mathrm{N}} \end{aligned}$ | $\circ$ $\infty$ $\infty$ $\infty$ |  | N |
| is | 命 | $\begin{aligned} & \infty \\ & \infty \\ & \stackrel{0}{\infty} \\ & \infty \end{aligned}$ | $$ | $\begin{aligned} & \text { H } \\ & \underset{\sim}{2} \\ & \stackrel{N}{2} \end{aligned}$ | $\begin{aligned} & \infty \\ & \infty \\ & \infty \\ & \infty \end{aligned}$ | $\begin{aligned} & \underset{~ N}{N} \\ & \stackrel{\rightharpoonup}{N} \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{n} \\ & \underset{\infty}{\infty} \end{aligned}$ | 冎 | ¢ | $\begin{aligned} & \text { N్ర } \\ & \text { ô } \end{aligned}$ |
| $8$ | $\underset{\sim}{i}$ | $\begin{aligned} & \stackrel{8}{\dot{\infty}} \\ & \underset{\sim}{\infty} \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{\alpha} \\ & \underset{\infty}{\infty} \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \underset{\infty}{\circ} \end{aligned}$ | $\begin{aligned} & \hat{N} \\ & \stackrel{\rightharpoonup}{\infty} \end{aligned}$ | $\begin{aligned} & \text { B } \\ & \text { N } \\ & \mathrm{N} \end{aligned}$ | $\underset{\substack{\stackrel{\rightharpoonup}{n} \\ \underset{\infty}{2}}}{ }$ |  | $\underset{\sim}{\aleph}$ | L2 ¢ $\infty$ |
| ONN | $\stackrel{0}{\bullet}$ | $\begin{aligned} & \text { N} \\ & \stackrel{N}{\mathrm{~N}} \end{aligned}$ | $\begin{aligned} & \text { O} \\ & \text { + } \\ & \text { N } \end{aligned}$ | $\begin{aligned} & \infty \\ & \underset{\infty}{\infty} \\ & \infty \\ & \infty \end{aligned}$ | $\begin{aligned} & \circ \\ & \underset{\sim}{\circ} \\ & \infty \end{aligned}$ | $\begin{aligned} & \text { n̄ } \\ & \stackrel{N}{N} \\ & \end{aligned}$ | $\begin{aligned} & \underset{\infty}{\infty} \\ & \underset{\infty}{+} \end{aligned}$ | $\begin{aligned} & \text { ๗̀ } \\ & \stackrel{\infty}{\infty} \end{aligned}$ | $\cdots$ | 8 Hi N |
| $8$ | $\begin{aligned} & \text { N } \\ & \text { No } \\ & \infty \end{aligned}$ | $\begin{aligned} & \circ \\ & \underset{1}{1} \\ & \text { ion } \end{aligned}$ | $\begin{aligned} & \infty \\ & \underset{\sim}{\infty} \\ & \infty \\ & \infty \end{aligned}$ | $\begin{aligned} & \text { H} \\ & \text { N } \end{aligned}$ | $\stackrel{\infty}{\stackrel{\infty}{+}}$ | $\begin{aligned} & \text { fín } \\ & \underset{\sim}{\circ} \end{aligned}$ | $\stackrel{\infty}{\text { O }}$ | $\stackrel{\text { ®̀ }}{\text { Ṅ}}$ | $*$ <br>  <br> ＋id <br> 0 | H゙ ¢ ¢ |
| $8$ | $\begin{aligned} & \text { H } \\ & \text { 人̀ } \end{aligned}$ | $\begin{aligned} & m \\ & \stackrel{m}{8} \\ & \infty \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{6} \\ & \stackrel{1}{6} \\ & \stackrel{1}{n} \end{aligned}$ | $\begin{aligned} & \text { H゙ } \\ & \text { ò } \\ & \text { o } \end{aligned}$ | $$ | $\begin{aligned} & 0 \\ & 0 \\ & 02 \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { N} \\ & \text { No } \\ & \text { 앙 } \end{aligned}$ | $\begin{aligned} & \text { L0 } \\ & \stackrel{N}{2} \\ & \stackrel{N}{N} \end{aligned}$ |  | $\stackrel{\square}{\infty}$ |

solution as shown in Table 2 indicates that the number of correct results is only one out of ten runs for each population size．The number of correct results is the counter of runs in which the correct value of objective function（fval＝764．57）is obtained．Also as the population size is increased there is no noticeable improvement in the number of correct results and the elapse time of each run increases especially for population size more than 100．The comparison of optimum values for constant population size （i．e．Pop．size $=100$ ）can be shown in Fig．2．It appears that the value of objective function of the first generation for
each run is different due to randomly initial population． Therefore，it can be concluded that the case of GAs optimization without initial population is unsuitable to the problem under consideration and it seems necessary to specify the initial population（i．e．initial point）．
In this study，the method of specifying initial population is adopted due to simplicity．To select initial population，a parametric study was carried out using 100 as population size and different set of initial values were chosen within bound limits of design variables，the following initial population was chosen as initial population for each case study in this search［ 1.81 .80 .20 .00010 .00010 .00010 .0001 0.00010 .00010 .0002327737280 .000232773728 ］．


Figure 1：comparison of results with constant population size（Pop size $=100$ ）of design example（without initial population）

To investigate the effects of initial population，population size，and establish the suitable population size，ten randomly runs were carried out for the example in the application process with initial population for each population size．Optimum values of these runs are showed in Table 4.
The comparison of the results of Tables 3 and 4 with and without initial population respectively indicates the effects of the initial population．In the case of solution without initial population and 1000 population size only one runs out of ten runs the correct result is obtained．While all in ten runs，the correct result is reached in the case of solution with initial population and only 100 population size．Also， it appears that as the population size is increased the number of correct result is increased．It seems that the optimum design solution of deign example can be achieved with initial population and 100 population size or more．

TABLE 4 COMPARISON OF 10 RANDOMLY RUNS WITH BASIC INITIAL POINT FOR VARIOUS POPULATION SIZE

| NN000 | Objective function（Total Price）UP |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 5 | 5 | § | § | 5 | 三 | $\equiv$ | § | $\xi$ |
| $\bigcirc$ | $\begin{aligned} & \stackrel{\infty}{\stackrel{1}{\infty}} \\ & \hline \end{aligned}$ |  | $\stackrel{-}{\circ}$ | $\begin{aligned} & \text { n } \\ & \infty \\ & \infty \\ & \infty \end{aligned}$ | $\begin{aligned} & \text { స్రి } \\ & \text { B } \end{aligned}$ | $\begin{aligned} & \circ \\ & +\underset{\infty}{+} \\ & \hline \infty \end{aligned}$ |  | $\stackrel{\underset{\infty}{\infty}}{\stackrel{\infty}{\infty}}$ | $\begin{aligned} & \infty \\ & \text { ભ. } \\ & \text { Mू } \end{aligned}$ | ¢ |
| 간 | $\begin{gathered} +\underset{\infty}{\infty} \\ \substack{\infty \\ \hline} \\ \hline \end{gathered}$ | $\begin{gathered} \underset{\infty}{\alpha} \\ \underset{\infty}{-\infty} \end{gathered}$ | $\begin{array}{r} \circ \\ \stackrel{+}{\infty} \\ \end{array}$ | $\begin{aligned} & \text { Ti } \\ & \text { No } \\ & \hline \end{aligned}$ | $\begin{array}{r} 7.0 \\ \infty \\ \hline 0 \end{array}$ | $\begin{aligned} & \text { H } \\ & \text { - } \\ & \hline \infty \end{aligned}$ | $\begin{gathered} \text { Y } \\ \underset{\infty}{\infty} \\ \hline \end{gathered}$ | $\begin{array}{r} 0 \\ \stackrel{j}{\infty} \\ \hline \infty \\ \hline \end{array}$ | N <br>  |  |
| ¢ | $\begin{aligned} & \text { O. } \\ & \dot{\infty} \end{aligned}$ | $\begin{array}{r} 10 \\ \infty \\ \substack{\infty \\ \hline \\ \hline} \end{array}$ | $\begin{aligned} & \underset{\infty}{9} \\ & \underset{\infty}{1} \end{aligned}$ | $\begin{array}{r} 0 \\ 0 . \\ \hline \infty \\ \hline \end{array}$ | $\begin{gathered} \underset{\infty}{\infty} \\ \underset{\infty}{2} \end{gathered}$ | $\begin{aligned} & \text { חֻ } \\ & \text { Co } \end{aligned}$ | $\stackrel{n}{\infty}$ | $\begin{array}{r} \infty \\ \stackrel{\infty}{\infty} \\ \stackrel{\infty}{\infty} \\ \hline \end{array}$ | $\underset{\sim}{\infty}$ | ने ¢ ¢ |
| g | $\begin{aligned} & \text { ח̛̣ } \\ & \text { +ín } \\ & \hline \end{aligned}$ | $\begin{aligned} & \infty \\ & \stackrel{\infty}{\infty} \\ & \infty \\ & \hline \infty \end{aligned}$ | $\underset{\sim}{\text { N }}$ | $\stackrel{N}{2}$ | $\begin{aligned} & \text { à } \\ & \stackrel{1}{\wedge} \end{aligned}$ | $\begin{aligned} & \text { N} \\ & \text { N } \\ & \end{aligned}$ | N | $\begin{aligned} & 0 \\ & \text { O} \\ & \vdots \end{aligned}$ | $\begin{aligned} & \text { gat } \\ & \text { of } \\ & \end{aligned}$ | 萵 |
| 옹 | $\begin{aligned} & \text { חְֻ } \\ & \substack{1 \\ \hline} \end{aligned}$ | $\begin{aligned} & \text { ơ } \\ & \stackrel{1}{1} \\ & \hline \end{aligned}$ |  | $\begin{aligned} & \text { No } \\ & \text { O } \\ & \hline \end{aligned}$ |  | $\begin{aligned} & \text { H } \\ & \stackrel{\infty}{\infty} \\ & \hline \end{aligned}$ | $\underset{N}{N}$ | $\begin{aligned} & \text { ח̛̣ } \\ & \stackrel{1}{\circ} \\ & \hline \end{aligned}$ |  |  |
| 8 |  | $\begin{gathered} \text { à } \\ \text { in } \end{gathered}$ | $\begin{array}{r} \text { H } \\ \text { B } \\ \hline \infty \end{array}$ | $\begin{aligned} & \infty \\ & \infty \\ & \\ & \hline \end{aligned}$ | $\widehat{\infty}$ | $\begin{aligned} & \text { n } \\ & \text { O} \\ & \\ & \hline \end{aligned}$ | $\begin{gathered} \text { Ni } \\ \text { N } \end{gathered}$ | $\begin{aligned} & \text { nín } \\ & \text { + } \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { ñ } \\ & \text { +í } \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { N } \\ & \text { ヘ̀ } \\ & \hline \end{aligned}$ |
| ค | $\begin{aligned} & \text { ח̛̣ } \\ & \text { +ín } \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { กí } \\ & \text { - } \end{aligned}$ |  | $\begin{aligned} & 0 \\ & 0.0 \\ & 0.0 \end{aligned}$ | $\underset{\sim}{2}$ | $\begin{aligned} & \text { חֻ } \\ & \text { N } \end{aligned}$ | $\begin{aligned} & \text { ón } \\ & \stackrel{y}{n} \end{aligned}$ | $\begin{aligned} & \text { חí } \\ & \text { +ín } \end{aligned}$ | $\stackrel{\text { ® }}{ }$ | $\stackrel{\substack{~}}{\stackrel{1}{\wedge}}$ |
| 8 | $\begin{gathered} N \\ \infty \\ \end{gathered}$ | $\underset{N}{N}$ | $\begin{aligned} & \text { no } \\ & \substack{1 \\ \hline} \end{aligned}$ |  | $\stackrel{\sim}{\infty}$ | $\begin{aligned} & \text { חֻ́ } \\ & \substack{ \\ \hline} \end{aligned}$ | ！ | $\stackrel{\substack{\text {＠}}}{\text { N }}$ | N | $\begin{aligned} & \text { ñ } \\ & \substack{\text { n } \\ \hline} \end{aligned}$ |
| 8 | $\begin{aligned} & \text { חo } \\ & \text { Oí } \end{aligned}$ | $\begin{aligned} & \text { ni } \\ & \substack{8 \\ \hline} \end{aligned}$ |  | $\begin{aligned} & \text { กo } \\ & \stackrel{1}{\wedge} \\ & \hline \end{aligned}$ | ＋ |  | 人̀ |  | R <br> ¢ <br> ¢ | ¢ |
| 8 | $$ | $\begin{gathered} \text { ne } \\ \substack{1 \\ \hline} \end{gathered}$ | $\begin{aligned} & \text { חof } \\ & \substack{1 \\ \hline} \end{aligned}$ | $\begin{aligned} & \text { ñ } \\ & \substack{1 \\ \hline} \end{aligned}$ | م | $\begin{aligned} & \text { ņ } \\ & \substack{1 \\ \hline} \end{aligned}$ | $\begin{gathered} \text { ne } \\ \substack{\circ \\ \\ \hline} \end{gathered}$ | ＋ | N | R <br> ¢ <br> ¢ |

## 5．VERIFICATION OF GAs OPTIMAZATION：

This is another optimization method．Sequential Quadratic Programming（SQP）method of gradient approach is used to verify the result of GAs optimization method using the same objective and constraints functions， input data of material properties，and same initial point，the final results coincide with the optimum solution using GAs optimization method as shown in Table 5.

Table 5：VERIFICATION OF RESULTS USING TWO OPTIMIZATION METHODS

| Symbol of <br> Variable | Gas | SQP |
| :--- | :--- | :--- |
| $x_{1}$ | 2.98 m | 2.98 m |
| $x_{2}$ | 2.98 m | 2.98 m |
| $x_{3}$ | 0.20 m | 0.20 m |
| $x_{4}$ | 1796.10 mm | 1796.03 mm |
| $x_{5}$ | 690.50 m | 689.20 mm |
| $x_{6}$ | 1796.10 mm | 1796.03 mm |
| $x_{7}$ | 1165.33 mm | 1165.27 mm |
| $x_{8}$ | 953.22 mm | 953.22 mm |
| $x_{9}$ | 338.88 mm | 338.90 mm |
| $x_{10}$ | 232.77 mm | 232.77 mm |
| $x_{11}$ | 232.77 mm | 232.77 mm |
| fval | 764.57 U. P． | 764.51 U．P． |

## 6．CONCLOSIONS

It is found that the genetic algorithms GAs optimization method is a suitable method that can be used to obtain the minimum cost（i．e．optimum design）of reinforced concrete box culvert．
It is important for any optimization problem using genetic optimization method to carry out the convergence studies to investigate the capability of establishing the optimum design with or without initial population and governing population size．
It can be noted for this study that the initial population of $\left[\begin{array}{lllllllll}1.8 & 1.8 & 0.2 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0.0001\end{array}\right.$ 0.0002327737280 .000232773728 ］（i．e．initial point）and population size of 100 give the correct results．While without initial population，there is no convergence even with high population size of 1000 ．

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[^0]:    1(Professor of Civil Engineering, Engineering College/Babylon University, Babylon, Iraq)
    2(Assistant Professor of Transportation Engineering, Engineering College/ Kufa University, Najaf, Iraq)
    3(B.Sc. of Civil Engineering, Engineering College/ Kufa University, Najaf, Iraq)

